JUNE 1966

REPORT

(Title -- Unclassified)

FINAL REPORT

GRAVITY GRADIENT STABILIZATION ELEMENTS
AND ANTENNA STRUCTURES MATERIAL STUDY

JUNE 1966

CONTRACT NAS 5-9599

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# GRAVITY GRADIENT STABILIZATION SYSTEM ELEMENTS AND ANTENNA STRUCTURES MATERIAL STUDY

## I. INTRODUCTION & OBJECTIVES

The Gravity Gradient Boom Program was conducted by the Marquardt Corporation under Contract NAS 5-9599, NASA Goddard Space Flight Center, with the object being to develop a welded, or joined, tube concept for eventual fabrication into an antenna type structure. Long thin lightweight structures have considerable potential for satellite application and are under extensive investigation for usage in several United States space programs. One of these applications is for gravity gradient satellite stabilization booms. Booms will provide attitude control when acted upon by earth's gravity as well as remain in predetermined orientation with respect to a specific earth surface area. In addition, if the booms are constructed of an electrically conductive material, the utilization can well be two-fold, namely, for both orbital stabilization and communication antenna.

Initial phases of the boom program required the design, development and fabrication of prototype antennas. Material recommendations from NASA were limited to Brush 25 (BeCu alloy) or equivalent. An equivalent material, (Brush 190) was selected by TMC because of availability in the form required and more attractive mechanical properties.

The final phase of the boom program required the optimization of boom design, the determination and implementation of manufacturing procedures, the mechanical property evaluation of a fabricated antenna section, (highlighting bend and torsional strength), and the design and fabrication of a deployment mechanism capable of projecting and/or coiling the completed configurated boom at 2 + 1/2 feet per second.



Included in the foregoing, were other specific NASA requirements for boom fabrication and control, which included extremely tight longitudinal straightness, flat edge faying surfaces, key quality assurance provisions encompassing center strip sectioning, surface condition, chemical-physical property documentation of each batch utilized, forming of half-sections, the assembly into modified tubular configurations and technique control for structurally joining the entire antenna length. Gas tight edge closures were not a requirement, although some consideration was given to that area.

#### II. SUMMARY

- 1. The strip, or foil, selected by TMC for slitting, forming, welding and perforating into completed boom antennas was Brush 190 in the  $\frac{1}{2}$  HM temper, 0.0015" thickness.
- 2. Procured strip width was 4" and utilized strip was a 2" wide section slit from the center of the strip.
- 3. Slitting of the foil was performed by Krusen Wire and Steel Company, Pico Rivera, California.
- 4. Forming of the strip was performed by Universal Molding Company, Lynwood, California, by using specially designed, precision ground rolls to induce longitudinal indentations (scoring) in order to improve the rigidity characteristics and preclude springback.
- 5. Resistance spot welding was performed by Inland Electronics, Pasadena, California and by TMC. The initial spot welding was of an experimental nature, and the second technique (TMC development) was a refinement of the former.
- 6. Perforation of the boom flanges was performed at the TMC Astro Shop facility.

- 7. An analytical study was conducted to generate the most feasible configuration for producing antennas capable of collapsing and being rolled onto a spool without plastic deformation, as well as reassuming its original tubular form upon deployment. (The final design configuration is shown in Fig. 4)
- 8. Prototype booms of three and ten foot lengths were fabricated and shipped to NASA.
- 9. Optimum (final) design booms of ten and forty-five foot lengths were fabricated and shipped to NASA. One length was installed on the deployment mechanism.
- 10. A deployment mechanism was designed, built and tested at TMC, using the final design forty-five foot booms to check its operation characteristics. Deployment speed was two feet per second. (Fig. 6)
- 11. Simultaneous bending and torsional tests, consisting of one foot pound and 0.005 ft. pound respectively, were performed on one foot sections of the boom final design. Results were not completely satisfactory.

  (Table I and Fig. 5)
- 12. Half-shell boom sections were passed through the forming rolls from one to three times.
- a. The single pass roll operation did not increase the bending-torsional property above 0.5 ft. lb.,
- b. The double pass roll operation increased the bendingtorsional property to 0.7 ft. lb.,
- c. The three pass roll operation (a post program endeavor) increased the bending-torsional properties to above the one ft. lb. requirement.

13. Storage tests were performed for periods up to one month.

Eight foot antenna lengths were tightly wrapped (flattened and compressed) on a 16" diameter spool and stored for periods of one week at room temperature.

At the end of each week, the antenna was deployed and rerolled a minimum of five times. Examination of the antenna surfaces and joined edges after each time period, indicated no detrimental effect.

#### III. RESULTS AND CONCLUSIONS

## A. Results

1. Figures 1 and 2 compare the results of room temperature tensile testing Brush 190 - 1/2 HM 0.0015" foil. Strain rate used was 0.001 in/in/sec. The data received from Brush is included for a direct comparison.

The TMC generated data revealed the following:

- a. Average proportional limit of 84.9 KSI
- b. Average modulus of elasticity  $16.7 \times 10^6$  psi
- c. Average ulimate tensile 133.4 KSI
- d. Average yield strength 126.7 KSI

As shown, the Brush data exhibits higher values for modulus and proportional limit but lower values in the significant yield area.

2. Fig. 3 is a photomicrograph detailing the microstructure of Brush 190 Beryllium-Copper strip, after the forming operation. During forming, scoring marks (indentations) are introduced longitudinally on the sheet surface, i.e. the "hat" section of the half shell. These scoring marks improve the basic rigidity of the alloy as well as preclude springback after the sheet passes through the rolls and is formed. Strain lines are introduced, however, there is no evidence of cutting although uniform thinning did result.

- 3. Fig. 4 depicts the proposed final boom configuration, which was determined after the analytical study was completed. All final design booms incorporated the basic dimensions shown. The dimensions were achieved with one pass through the forming rolls. Subsequent two and three passes through the rolls did not disturb or alter the configuration, although the improvement in bending strengths exhibited was very significant.
- 4. Fig. 5 is a photograph of the initial torsional bending tests. Bending loads were one ft. lb. and torsional loads 0.005 ft. lbs. applied simultaneously. The test specimen is one foot long and of final antenna design. Test results on this single roll pass specimen are tabulated in Table I. The antenna passed the parallel to perforated edge tests but failed the transverse tests. Later tests of an identical nature were performed on double and triple rolled welded-perforated antennas. The double rolled antennas achieved 0.7 ft. lb. transversely before failure and the triple rolled antennas successfully passed the one lb. bending 0.005 torsional moments.
- 5. Figure 6 is a photograph of the TMC designed and built boilerplate deployment mechanism. This deployment mechanism was successful in satisfactorily coiling up and deploying, as required, forty-five foot lengths of antenna. Maximum deployment speed was two feet per second.

#### B. Conclusions

Based on the results of this program, it may be concluded:

- 1. The Brush 190 1/2 HM temper, 0.0015" thick foil is suitable both from a strength and weight standpoint for fabricating antenna half-shells.
- 2. The forming technique, whereby longitudinal scoring of the formed surfaces increases the rigidity and precludes springback, is a feasible approach.

- 3. The spot welding of the half-shells into a semi tubular configuration is satisfactory for reliably joining relatively long antenna lengths.
- 4. The multi-pass, or reroll technique, to improve the bending-torsional moment capability was a major break-through in assuring structural capability.
- 5. It is possible to design and build a deployment mechanism capable of quickly and reliably extending or rewinding the boom antenna as required.
- 6. That the basic development techniques worked out in this program can be extrapolated into a more sophisticated approach for producing boom antenna at lengths to 1000 feet or greater.

## IV. RECOMMENDATIONS

The experience derived from this program enable recommendations to be separated into five categories, as follows:

- 1. Additional design and development work is required to insure that a <u>single</u> roll forming pass will generate adequate rigidity in the Brush 190, i.e. one foot lb. bending and 0.005 torsional moments.
- 2. Efforts are required in the fabrication phase to permit the coiling of extremely long lengths of half-shell antenna for subsequent joining, perforating and recoiling as completed antenna with a minimum space utilization.
- 3. The deployment mechanism requires additional design activity to reduce its size and weight for eventual space application. In addition, greater sophistication of its operational features are required.
- 4. The joining of the flanged boom half-shells should be re-evaluated, with the prospect of using the Laser welding process for gas closure seaming. The Laser process has extreme potential if the development effort is adequately supported.

5. Serious consideration should also be given to modifying existing Electron Beam welding equipment to continuously seam the antenna flanges for gas tight closure. The technique of application should incorporate a method for both localizing the controlled protective vacuum and positioning the tooling outside the chamber. Both modification of existing E. B. equipment and the design of development tooling will be required.

#### V. TECHNICAL DISCUSSION

This program was divided into three main phases:

- A. Analytical Studies
- B. Material & Process
  - 1. Boom Configuration (Prototype)
  - 2. Forming Operation (Final Boom Configuration)
  - 3. Testing
    - a. Straightness
    - b. Stiffness
    - c. Storage & Flexure
- C. Deployment Mechanism Design
- A. Analytical Studies (See Attachment)

Analytical studies were performed to determine the forces, moments and displacements when loadings are applied to fully flatten an optimum tubular cross section. Specific boom geometry requirements included stiffness, mass, width and no plastic deformation when flat and coiled.

#### B. Material & Process

The boom material was required to be a copper-beryllium alloy,

Brush 25, or equivalent, and to meet the requirements of Brush Beryllium Company Data

Sheet on Brush 25. Alloy(s) utilized were required (end item usage) to be rolled to a

width of 4-6" and the section for antenna fabrication to be extracted (slit) to a two inch maximum width from the center section of the starting strip. Maximum crossbow permissible (vertically) over the two inch width was 0.125". Additional requirements for the selected post slit material and fabricated antennas included a maximum deviation of 0.25" for every six feet, a maximum permissible edge ripple of 0.010" amplitude with a six inch minimum period and the surfaces to be clean, free of oxides, marks and scratches. Deliverable antennas were to be in three foot, ten foot and forty-five foot lengths.

Marquardt selected Brush Beryllium 190 over the Brush 25 alloy, because while these alloys are essentially the same, the 190 can be obtained in the mill heat treated condition, which eliminates the need for post fabrication heat treatment. Heat treating after fabrication would be difficult because of the absence of available facilities or by the inducement of warpage in the finished product.

The formability of Brush 190 is less than that of Brush 25, a problem largely overcome by the thinness of the antenna material and the permissible relatively large radius of formed half sections. Material thickness limitations were imposed by the maximum permissible mass per unit length of 0.6 x 10<sup>-3</sup> slug/feet.

Brush 190 beryllium-copper of 1/2 HM temper has a strain of 0.0045 in/in at the proportional limit, and 0.008 in/in at the yield strength.

(Analytical studies conducted prior to final boom configuration determination indicated that the strain, when flattened and coiled, was 0.0037 in/in, which permitted some margin during manufacture.) See Figs. 1 and 2 for a comparison of Marguardt and Brush Beryllium data.



## 1. Boom Configuration (Prototype)

form of material utilized to be foil or strip. This strip material was to be formed into a modified tubular configuration and capable of being flattened and coiled.

Permissible radius on the finished tube could vary from 0.35" for a wall thickness of 0.002" to a radius of 0.50" for a wall thickness of 0.0015". Maximum unrestrained tube dimension, including flanges, was not to exceed two inches.

Areas of prime consideration for the completed antenna were weight, bending and torsional strengths, thickness and resultant microstructure, as well as the overall techniques of manufacture, which specifically highlighted forming and joining.

The numerous joining processes available were evaluated for application to the antenna. This evaluation encompassed TIG and EB welding, Laser welding, resistance welding (both spot and seam), brazing and soldering and solid state diffusion bonding. Marquardt selected resistance spot welding as the most applicable process. Seam welding was discarded because gas tight closures were not a specific requirement. TIG and E. B. welding were eliminated because of weld shrinkage and the excessive amount of cast structure generated. Laser welding was rejected because the "state-of-the-art" of this process had not sufficiently progressed for serious consideration as a reliable technique. Brazing and soldering were deemed unsuitable because of the necessary long time development program needed and the problems of reliably joining the long lengths without undue distortion. Diffusion bonding was disallowed because of the difficulty in structurally joining the thin pieces of material, as well as the rather experimental nature of the process.

For the initial, or prototype, forming of antenna configurations, several pounds of 1/2 HM temper Brush 190, 0.0015" thick and 1-5/8" wide wide were procured. Although the analytical study leading to final boom



configuration was not yet completed, it was felt necessary to generate additional data on the as-formed characteristics of Brush 190. (Preliminary calculations from the analytical study had accounted for the stresses induced by flattening and coiling of the anticipated shape and indicated an optimum cross section could be satisfactorily produced that would not plastically deform.) Prototype forming equipment was designed, built and utilized for producing initially two three-foot prototype booms. Half sections were produced by passing the copper-beryllium strip through the forming rollers. The forming rolls were designed to induce multiple scoring (indentations) on the crown surface (hat) of the half-shell to increase the rigidity of the structure and preclude springback of the Brush 190 foil. After forming, the half-sections were resistance spot welded into the prototype boom configuration. Forming was performed by the Universal Molding Company, Lynwood, Calif., and spot welding was performed by Inland Electronics, Pasadena, Calif. Because this was a prototype configuration, torsional and bending tests were not conducted, and the booms were subsequently shipped to NASA.

To determine the effects of the forming on the microstructure of the foil, a metallographic examination was conducted. Figure 3 shows the microstructure of one of the indentations. As noted, a large number of strain lines were introduced, however, there is no evidence of cutting and the indentations merely show uniform thinning.

## 2. Forming Operation (Final Boom Configuration)

a. Detailed consideration of necessary manufacturing sequences was given to determine the order of operation for producing completed booms. The sequence of operations were determined to be  $(\underline{1})$  slitting,  $(\underline{2})$  forming,  $(\underline{3})$  spot welding, and  $(\underline{4})$  perforating.



pleted and the final boom configuration determined. The configuration is depicted in Fig. 4. This configuration, when flattened and coiled, has 0.0037 in/in strain induced well within the capability of the 1/2 HM Brush 190 without plastic deformation occurring. It was later determined that the bending and torsional requirements of the completed boom were within the scope of this configuration.

Thirty pounds of Brush 190 alloy were received from the Brush Beryllium Corporation, custom rolled to 4" width, 0.0015 + 0.0001 in thick and of 1/2 HM temper. A strip 1.850" wide was slit from the center. Slitting was performed by Krusen Wire and Steel Company, Pico Rivera, Calif., a local material vendor-representative of Brush. Krusen utilizes a precise technique to eliminate or leave edge ripple to a minimum. The slit material was formed into half-shells by Universal Molding Company, Lynwood, Calif., who had previously formed the prototypes. (The forming rolls utilized for the prototype antennas had been reworked and precision ground to achieve the requirements of the optimum boom configuration.) Universal Molding is a specialist in the field of roll forming and their preliminary studies on shaping the beryllium-copper foil revealed that it was impossible to form the required shape using standard techniques. The spring back and thinness of the foII required that it be formed around radii of 5-10 times the thickness of the material, and the most feasible method of achieving this was by scoring or inducing longitudinal indentations. The two rolling cylinders were designed to induce the necessary indentations, with depth and spacing variations permitted by adjusting the rolling pressures. This technique was used to generate the tubular half-sections, with the scoring induced into the "hat" area alone. The optimum design incorporated two roll passes.

d. The half-shells formed by Universal Molding were resistance spot welded by Inland Electronics into the optimum configuration. Each completed antenna was ten feet long. Following welding, the booms were perforated at the Marquardt shop facilities and subsequently shipped to NASA. (Later on in the program, following the testing phase, two forty-five foot long antennas were formed, welded and perforated as above with one being installed on the deployment mechanism and shipped to NASA.)

## 3. Testing

As noted, the two ten foot antennas shipped to NASA were of the final, or optimum, design. Manufacturing techniques utilized final forming, tooling, welding procedures and perforation methods applicable for forth-five foot antennas.

A limited amount of testing was required to determine specific properties, such as longitudinal straightness, stiffness, storage and flexure.

a. The longitudinal straightness of the flanged edges were not to exceed  $\pm 1/4$ " in any three foot length, nor the rotation of the seam exceed  $30^{\circ}$  in a forth-five foot length. These requirements were met in the shipped forty-five foot antenna lengths.

b. The boom material was required to sustain, without collapsing, a one-ft-lb bending moment during the simultaneous application of a torsional moment of not less than 0.005 ft-lb. Preliminary bending and torsional testing was performed on three one-foot sections of the final antenna design. The sections were mounted between two plexiglass sheets using an epoxy cement, as shown in Fig. 5. Specimens were loaded mechanically, in a transverse direction to a moment of one-foot pound, with a torque of 0.005 ft-lb simultaneously introduced.



This was accomplished by calculating the lever arm required, i.e., the distance from the centerline of the specimen, and loaded accordingly. No plastic deformation occurred under the one ft-lb loading with the 0.005 ft-lb torsional also applied, when loaded in a direction parallel to the perforated edges. Failure did occur when the same loadings were applied transversely. Results are shown in Table I. (These tests were initially performed on single roller pass final design antennas, and retested on the double roller pass final design.)

The results of bending and torsional testing the double pass final design antennas indicated a capability to sustain a bending moment of 0.7 ft-lb before collapsing, not quite within specification requirements. Technique of applying the bending and torsional loads were similar to the initial tests. At a later date, triple roll pass specimens were evaluated and found to be capable of sustaining the one ft-lb bending and the 0.005 ft-lb torsional moments without failure. However, this latter development was achieved too late for the forming technique to be utilized on the prior shipped forty-five foot antennas. However, a ten foot triple rolled length was fabricated and sent to NASA for further testing.

c. A finished (completed) boom of final design three foot minimum length was required to be coiled on a reel and stored for a period of one week, removed, released and rerolled a minimum of five times. The storage and testing operation was performed on a weekly basis for a period of one month. The joined edges and surface smoothness were to be visually examined each week. All NASA requirements were met and exceeded. Boom sections (eight feet in length) were fabricated and wrapped around sixteen inch diameter spools, similar to the one used on the Deployment Mechanism (see Section C) and subjected to the storage, release and reroll test requirement. Inspection of the joined edges and surfaces at the intervals required showed all boom lengths thus tested to be satisfactory.

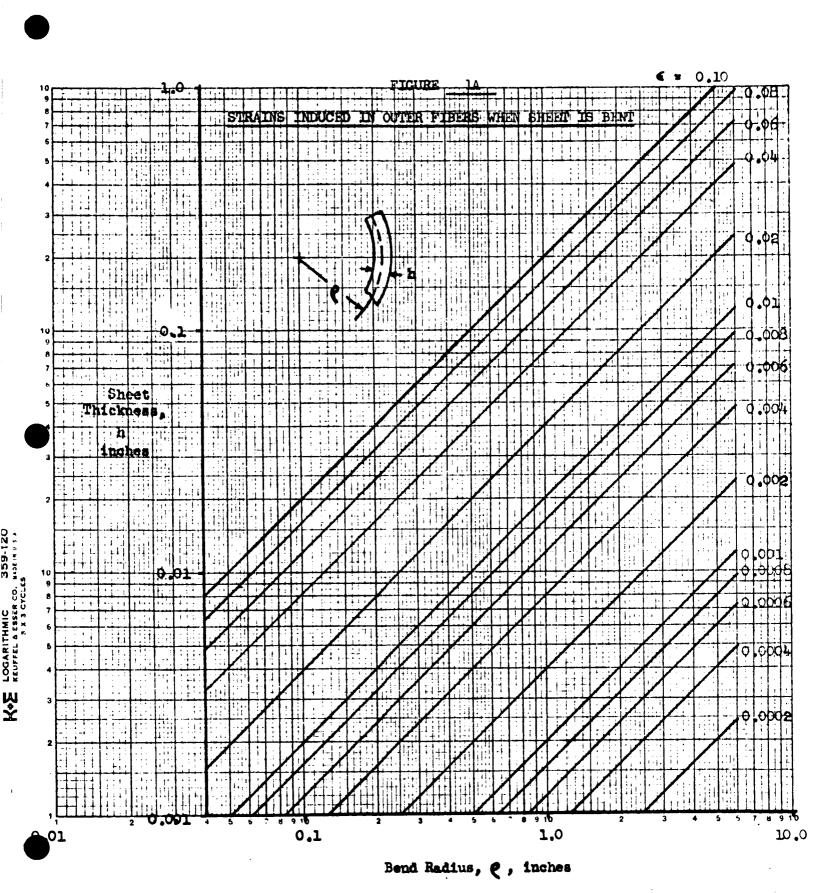
### C. Deployment Mechanism

In order to achieve all goals of the gravity gradient boom program, it was necessary to design and build a device capable of deploying the fabricated antennae. This device, termed a deployment mechanism, was required to deploy a forty-five foot antenna at the rate of  $2 \pm 1/2$  feet per second. The deployment mechanism could be of the boiler plate/laboratory type and should be able to deploy the boom in a vertical direction, such that the antenna at thirty feet would not deviate by more than  $\pm 1/2$  inch from the vertical.

A motor driven deployment mechanism was designed (and built by Marquardt) with a maximum speed of two feet per second. Guide rollers were employed between the reel and sprocket drive to allow the antenna to spring out to its tubular configuration in a uniform and controlled manner. Deployment stopping was achieved by incorporating a revolution counter on the storage reel. When the forty-five foot antenna is deployed, the counter activates relays to disengage the clutch driving the sprocket and applies the electromagnetic brake to the storage reel shaft. All components for the deployment mechanism (motor, speed, control, gear reducer, sprocket, electromagnetic clutch and brake, overrunning clutch, pulley, etc.) were selected as off the shelf stock items.

The type of perforations used in each antenna was determined after a careful investigation. It was considered desirable to select a standard commercial type perforation configuration to minimize the problems (cost and ease) in the procurement of matching sprockets. Eventually, the EIA R8-227 perforation, which consists of 0.046 inch diameter holes on 0.100 inch centers, was selected.

All antennas fabricated by Marquardt incorporated the foregoing perforations. Actual perforations were performed in the Marquardt Manufacturing
facility. Fig. 6 depicts the deployment mechanism with a section length of antenna
installed.



-1-

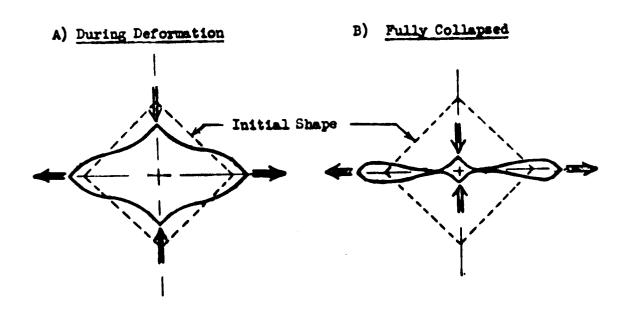


FIGURE 2A: DEFORMATION SEQUENCES OF DIAMOND-SHAPE CROSS SECTION

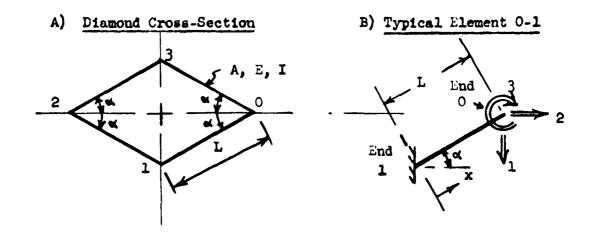


FIGURE 3A

The geometry of a diamond-shaped cross-section boom is illustrated in Figure 2A. The rigid jointed truss is composed of the four identical members 0-1, 1-2, 2-3, and 3-0. The length, area, moment of inertia, and Modulus of Elasticity of each member is designated by the symbols L, A, I, and E, respectively.

Loads applied at the apexes of the truss are illustrated in Figure 2A. Because of the symmetry of both stiffness and loading the elastic analysis of the structure is somewhat simplified. In fact the deformation of the structure may be evaluated by considering the motion of a single member. For this purpose the cantilever beam member 0-1, shown in Figure 3A (b), is selected. For small elastic strains the deformations  $U_1$ ,  $U_2$ , and  $U_3$  induced by the loads  $F_1$ ,  $F_2$ , and  $F_3$  are related by the matrix expression,

$$U = AF \qquad (A.1)$$

Matrix (A.1) represents the expression

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}$$
(A.2)

where

$$A_{11} = \frac{L^{3}}{3 \text{ EI}} (\cos^{2}\alpha + \frac{3 \text{ I}}{A \text{ L}} 2 \sin^{2}\alpha)$$

$$A_{12} = \frac{L^{3}}{3 \text{ EI}} (1 - \frac{3 \text{ I}}{A \text{ L}} 2) \sin \alpha \cos \alpha$$

$$A_{13} = \frac{L^{2}}{2 \text{ EI}} \cos \alpha$$

$$A_{22} = \frac{L^{3}}{3 \text{ EI}} (\sin^{2}\alpha + \frac{3 \text{ I}}{A \text{ L}} \cos^{2}\alpha)$$

$$A_{23} = \frac{L^{2}}{2 \text{ EI}} \sin \alpha$$

$$A_{33} = \frac{L}{2 \text{ EI}}$$

Because of symmetry the change in slope at Station 0 is zero. If use is made of this boundary condition one obtains from (A.2) the reduced matrix expression

The coefficients B of (A.3) are

$$B_{11} = A_{11} - A_{13}^{2}/A_{33}$$

$$= \frac{L^{3}}{12 \text{ EI}} \left( \cos^{2} \alpha + \frac{12 \text{ I}}{A \text{ L}^{2}} \sin^{2} \alpha \right) \qquad (A.4)$$

$${}^{B}_{12} = {}^{A}_{12} - {}^{A}_{13}{}^{A}_{23}{}^{/A}_{33}$$

$$= \frac{L^{3}}{12 \text{ EI}} \left( 1 - \frac{12 \text{ I}}{A \text{ L}^{2}} \right) \sin \alpha \cos \alpha \qquad (A.5)$$

$$B_{22} = A_{22} - A_{23}^{2}/A_{33}$$

$$= \frac{L^{3}}{12 EI} \left( \sin^{2} \alpha + \frac{12 I}{A L^{2}} \cos^{2} \alpha \right) \qquad (A.6)$$

$$F_{3} = - (A_{3}/A_{3}) F_{1} - (A_{23}/A_{33}) F_{2}$$

$$= - \frac{L}{2} (F_{1} \cos \alpha + F_{2} \sin \alpha) \qquad (A.7)$$

# FORCES, MOMENTS, AND DISPLACEMENTS

If use is made of the previously derived expressions the axial and shearing forces and the bending moments and deflections may be computed. The following results are obtained,

$$T = Axial tension = -F_1 sin \alpha + F_2 cos \alpha$$

$$V = Shearing force = F_1 cos \alpha + F_2 sin \alpha$$

$$M = Bending Moment = (L/2 - x)(F_1 cos \alpha + F_2 sin \alpha)$$

$$U_1(x) = Vert. Deflection = \frac{x^2(3 L - 2 x)(F_1 cos \alpha + F_2 sin \alpha)}{12 EI}$$
(A.8)

## FORCES, MOMENTS, AND DISPLACEMENTS

cont'd.

For the present investigation we are primarily interested in the stresses induced in the structure while in the process of being deformed. It is therefore convenient to express V, M, and  $U_1(x)$  in terms of the vertical displacement of the ligament end,  $U_1(L)$ . From (A.8) one obtains the following relations

$$V = 12 \text{ EI } U_{1}(L)/L^{3}$$

$$M = \frac{6 \text{ EI }}{L^{3}} (L - 2 \text{ x}) U_{1}(L) \qquad (A.9)$$

$$U_{1}(x) = (x/L)^{2}(3 - 2 \text{ x/L}) U_{1}(L)$$

where

$$U_1(L) = \frac{L^3}{12 EI} (F_1 \cos \alpha + F_2 \sin \alpha)$$
 (A.10)

The outer fiber strains induced by bending will next be determined. If use is made of the second of Eqs.(A.9) one obtains

$$\epsilon_b = \pm \frac{C}{L3} (L - 2 x) U_1(L)$$

$$\pm \frac{6 C}{L3} (L - 2 x) U_1(L) \qquad (A.11)$$

then

Eqs.(A.11) are for a flat sheet of thickness, h. Tensile strains at the outer fiber are considered positise.

# DISPLACEMENTS WHEN CROSS-SECTION IS FULLY COLLAPSED

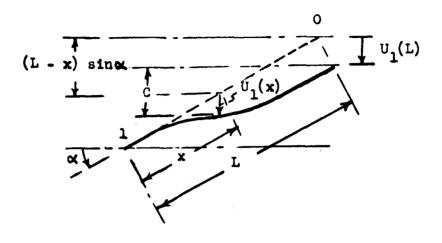


FIGURE 4A

Figure 4A illustrates the deformation patterns which the structure will undergo when loads are applied at the four corners of the cross-section. It is seen that the members cannot be fully flattened before they will make contact at some station along their length. Once the contact has been made to further flatten the structure will induce large strains at the four corners. However, it is possible to collapse the cross-section nearly flat while keeping the strains within the elastic range. This section will determine this permissible flatness.

From Figure hA the vertical clearance, C, is seen to be given by the expression,

$$C = (L-x) \sin \alpha + U_1(x) - U_1(L)$$
 (A.13)

Substituting  $U_1(x)$  from the last equation of (A.9), into (A.13) gives

$$C = (L-x) \sin \alpha + [(x/L)^2(3-2x/L) - 1]U_1(L)$$
 (A.14)

At the time contact is made the clearance is zero. From (A.14) one obtains the displacement  $U_1^0(L)$  at zero clearance as

$$\frac{U_1^0(L)}{L \sin \alpha} = -\frac{(1 - x/L)}{3(x/L)^2 - 2(x/L)^3 - 1}$$
 (A.15)

Within the range of , 0 < x/L < 1, two roots will give the maximum values for (A.15). These roots are at x/L = 0.25 and 1.0. Since the last of the two roots is a boundary condition only the value x/L = 0.25 is of meaning here. Inserting this root into (A.15) one obtains

$$\frac{U_1^0(L)}{L \sin \alpha} = 0.889 \tag{A.16}$$

Next, substituting (A.16) into (A.14) one obtains the clearances at other stations along the member as,

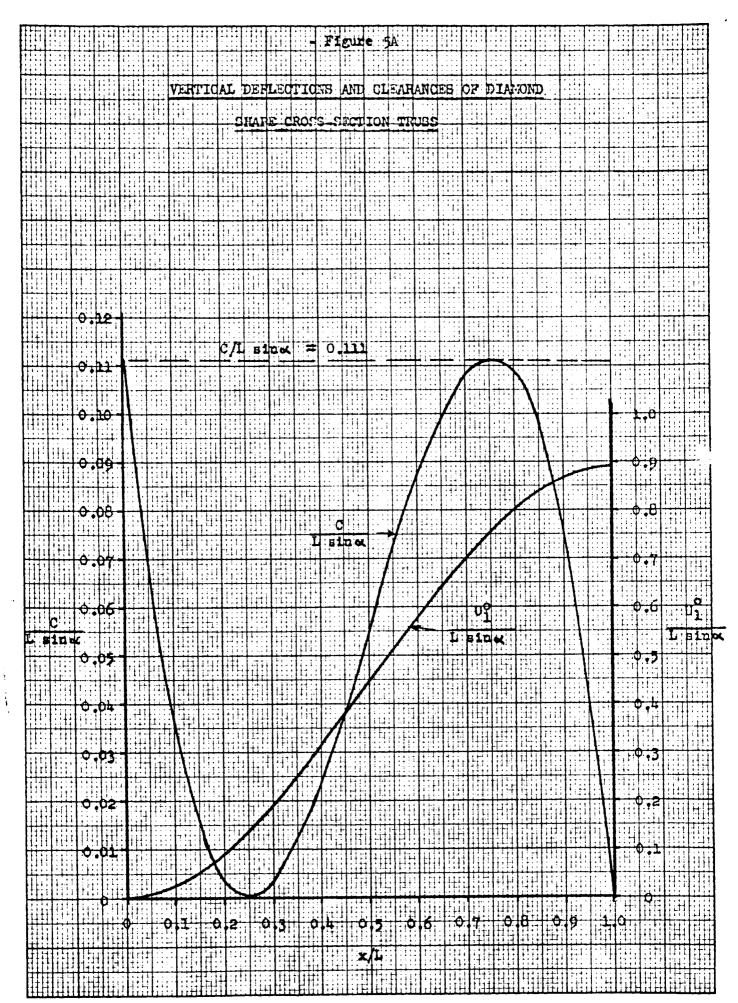
$$\frac{C^{\circ}}{L \sin \alpha} = 0.111 - x/L + 0.889(x/L)^{2}(3 - 2 x/L)$$
 (A.17)

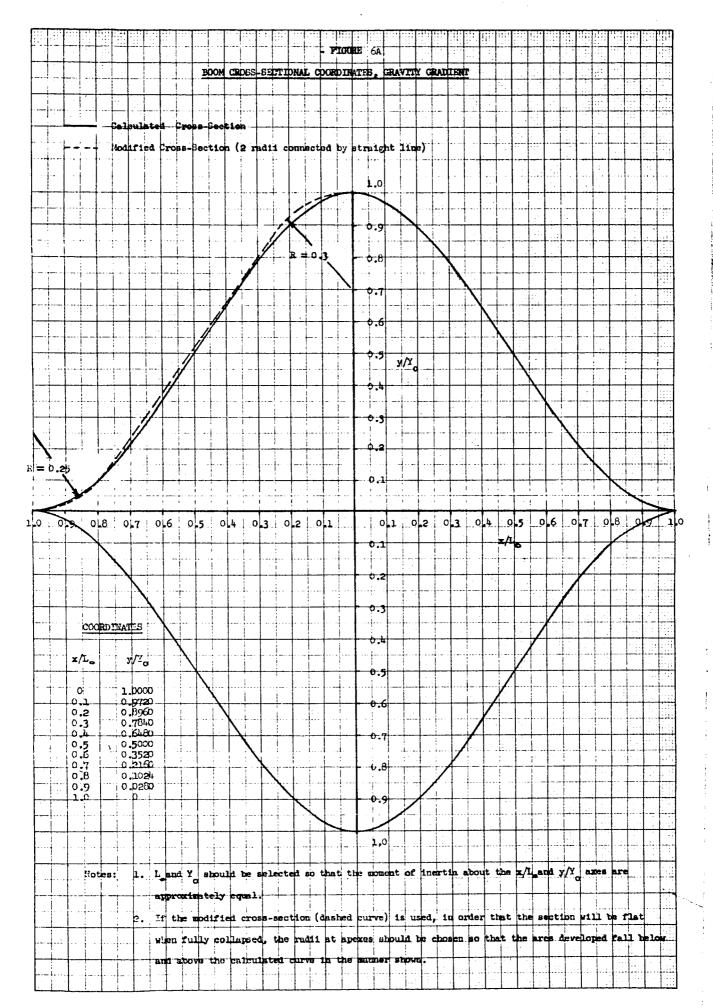
If (A.16) is substituted into (A.12) the maximum bending strains which are developed are obtained. These are found to be

TABLE A-1: CALCULATION OF VERTICAL DEFLECTIONS AND CLEARANCES OF

DIAMOND SHAPE CROSS-SECTION TRUSS

(1)	(2)	(3)	(4)	(5)	(6)	(7)
x/L	2 x/L	(x/r)2	3 -(2)	0.889 x (3)	$U_1^{o}(x)/L$ since	C/L sin &
					(4) x (5)	0.111 -(1)+(6)
0	0	0	3.00	0	0	0.111
0.1	0.2	0.01	2.80	0.00389	0.02489	0.0359
0.2	0.4	0.04	2.60	0.03556	0.09246	0.00346
0.25	0.5	0.0625	2.50	0.05563	0.13908	0
0.3	0.6	0.09	5.40	0.08001	0.19202	0.00302
0.4	0.8	0.16	2.20	0.14224	0.31293	0.0239
0.5	1.0	0.25	2.00	0.22225	0.44450	0.0555
.0.6	1.2	0.36	1.80	0.32004	0.57607	0.08707
0.7	1.4	0.49	1.60	0.43561	0.69698	0.10798
0.8	1.6	0.64	1.40	0 <b>.56</b> 896	0.79794	0.10894
0.9	1.8	0.81	1.20	0.72009	0.86411	0.07511
1.0	2.0	1.00	1.00	0.889	0.889	0





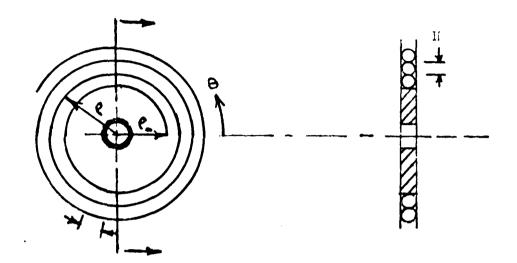


FIGURE 7A

Figure 7A shows an idealized concept of the way in which a string, cable, etc., might be coiled on a drum. To determine the drum size required one may treat the coiled member as a spiral having a constant rise per revolution. For this investigation the rise is denoted by the symbol, H. The radius of curvature, Q, is then given by the expression

$$\ell = \ell_{\circ} \left[ 1 + H \theta / 2\pi \ell_{\circ} \right]$$
 (B.1)

where  $\ell$  is the radius of curvature at the point where the coiling begins and  $\theta$  is the coiling angle measured also from the initial point.

The arc length, S, is given by the relation

$$S = \int_{0}^{S} [e^{2} + (de/de)^{2}]^{1/2} de \qquad (E.2)$$

Performing the mathematics required with (B.1) and substituting the results into (B.2) gives,

$$S = e_0 \int_0^S \left[ 1 + (b/2)^2 + b + (b/2)^2 e^2 \right]^{1/2} d\theta \qquad (B.3)$$

where  $b = II/\pi \rho_0$ 

Integrating (B.3) and evaluating between the limits indicated gives the arc length as,

$$S = \ell_0 \left\{ (1/2 c) \left[ (1 + c \theta) \left( (1 + c \theta)^2 + c^2 \right)^{1/2} - (1 + c^2)^{1/2} \right] + (c/2) \ln \left( \frac{1 + c \theta}{1 + (1 + c^2)^{1/2}} + \frac{c^2}{1 + c^2} \right)^{1/2} \right]$$
where
$$c = H/2\pi \ell_0$$
(B.5)

...q. (B.4) is somewhat cumbersome to evaluate. In certain problems it may be used in a more compact form with adequate accuracy. For instance, the problem where c << 1 may be evaluated from the relation,

$$S = \ell_0 \left\{ \theta(1 + c \theta/2) + (c/2) Ln (1 + c \theta) \right\}$$
 (B.6)

When  $\theta < \ell_0/2$  H then (B.6) reduces still further. The result in this case is the well known relation

$$s = \ell_0 \theta \qquad (\nu_{\cdot,i})$$

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Specimen No.	Loading Direction*	Length	Load	Results
ı	P	12 in.	1.0 16.	Pass
1	Т	12 in.	0.5 lb.	Shear Failure
2	P	12 in.	1.0 lb.	Pass
2	T	12 in.	0.45 16.	Shear Failure
3	P	12 in.	1.0 lb.	Pass
3	T	12 in.	-	Not tested

<sup>#</sup> P = parallel to perforated edges
T = transverse to perforated edges

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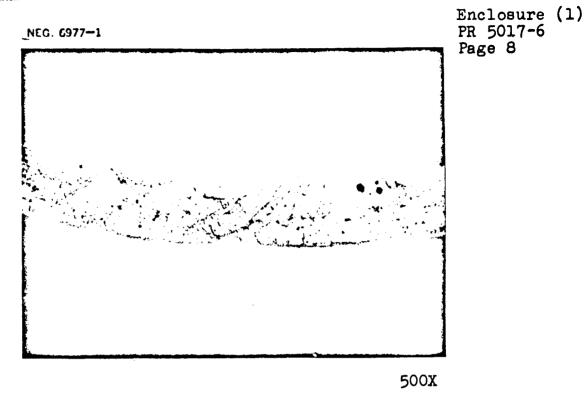
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Enclosure (1)

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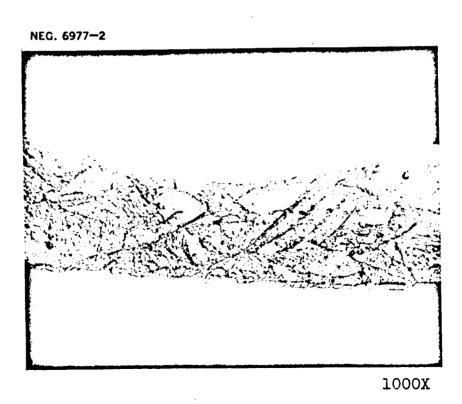
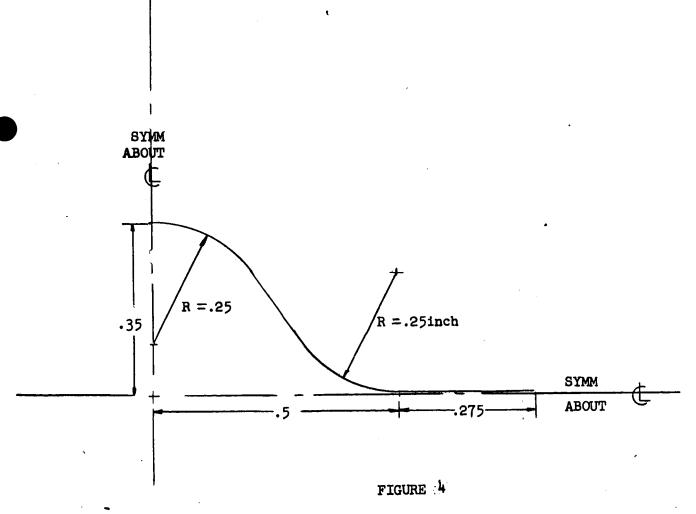
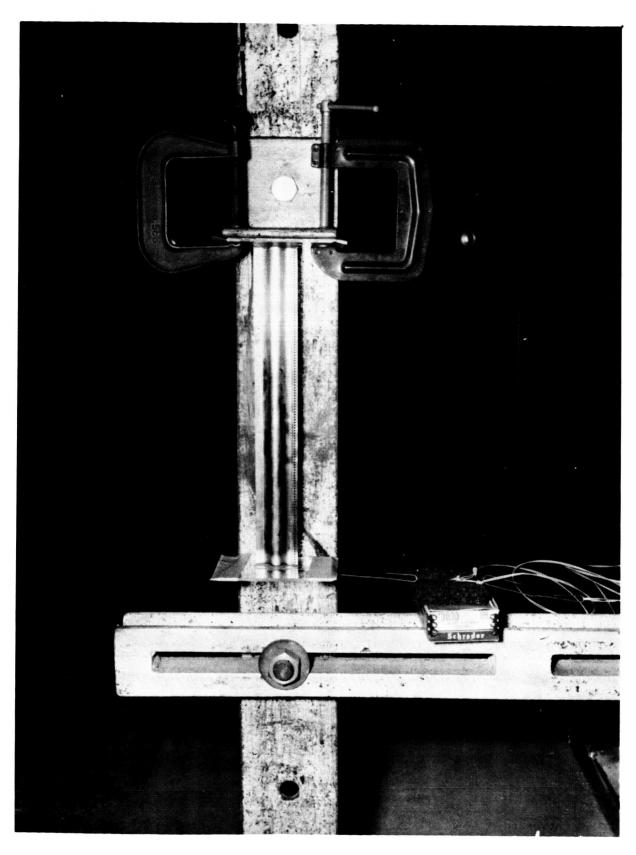


Figure 3. Microstructure of Brush 190 Beryllium-Copper Strip After Forming in the Mill Heat Treated Condition.

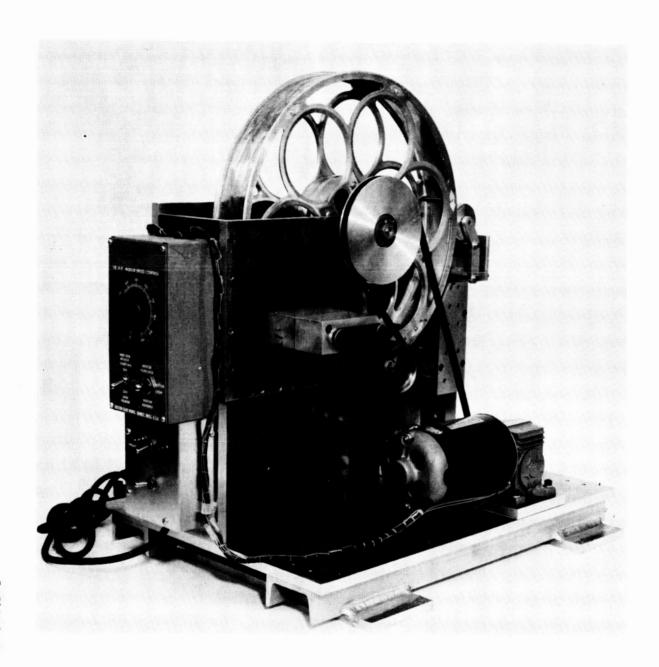
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PROPOSED FINAL BOOM CONFIGURATION



SETUP FOR BENDING--TORSIONAL TEST ON ONE-FOOT ANTENNA 4 MARCH 66 (U)



BOOM DEPLOYMENT MECHANISM -- BOILERPLATE 10 MARCH 66 (U)

FIGURE 6

